

Title: Using Right Triangles

Brief Overview:

The students will be able to apply the use of the Pythagorean Theorem and special right triangles to answer questions based on data collected from the Internet.

Links to NCTM 2000 Standards:

- **Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation**

These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

- **Geometry and Spatial Sense**

The students will use the Pythagorean Theorem and special right triangles to solve real-world problems.

- **Measurement**

The students will use the Internet to convert longitude and latitude coordinates into miles.

- **Data Analysis, Statistics, and Probability**

The students will use the Internet to collect data to research a particular hurricane and to find the heights of buildings.

Links to Virginia High School Mathematics Core Learning Units:

- **G.7**

The students will solve practical problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry. Calculators will be used to solve problems and find decimal approximations for the solutions.

- **A.13**

The students will estimate square roots to the nearest tenth and use a calculator to compute decimal approximations of radicals.

- **A.14**

The students will solve quadratic equations in one variable both algebraically and graphically. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

Links to National Science Education Standards:

- **Earth and Space Science**

Students will research a hurricane and use the data to answer real world questions.

Links to Virginia High School Science Core Learning Units:

- **ES. 3**

Students will investigate and understand how to read and interpret maps, direction and distance measurements on a map, and location by latitude and longitude.

Grade/Level:

Grades 7-12; Geometry

Duration/Length:

2-3 90-minute periods

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Simplifying expressions with radicals
- Using the Pythagorean Theorem
- Multiplying and dividing fractions, along with other operations involving fractions.

Objectives:

Students will be able to:

- apply the Pythagorean Theorem and the properties of special right triangles.
- access the Internet to gather information on past hurricanes and building heights.
- answer questions using the Pythagorean Theorem based on the data collected.

Materials/Resources/Printed Materials:

- Computers with Internet access
 - Websites for Worksheet #3 (Hurricanes)
 - <http://www.usatoday.com/weather/hurricane/whhistory.htm>
 - www.indo.com/distance/
 - Websites for Worksheet #5 (Shadows)
 - <http://www.nps.gov/wamo/monument/monument.htm>
 - <http://www.worldstallest.com/hot500.html>
 - <http://www.imsa.edu/~mozart/fun/needle.html>
 - <http://www.kyes-world.com/pythagor.htm>

- Calculators
- Activity sheets (one per group)

Development/Procedures:

1. The teacher will review the Pythagorean Theorem and the rules of special right triangles.
2. The teacher will use Worksheets #1 (30-60-90) and #2 (45-45-90) to make sure the class understands properties of special right triangles.
3. The teacher will have the class divide into groups of 2-3 students giving each group a set of activity sheets.
4. The teacher will allow groups to work on the Internet to collect data for Worksheet #3 (Hurricanes) and for Worksheet #5 (Shadows). This will allow students to complete all Internet research at one time.
5. After returning to class, the teacher will have the groups complete Worksheets #4 and #5 using the data collected from the Internet.

Assessment:

Answer keys for Worksheets #1, #2, and #5 follow those pages. For Worksheets #3 and #4, students' answers will vary based upon the hurricane selected. Therefore, a rubric follows those pages for the teacher's guidance. The teacher can assess the students' understanding using Worksheet #1 and #2 on the special right triangles. The teacher may also use the summary that is written on Worksheet #4 to see if students have an understanding of the activity. Following Worksheet #4, a rubric is also provided to the teacher and to the students.

Extension/Follow Up:

A teacher may choose to use part of the activity as an extension or follow-up. A teacher may use the worksheets involving hurricanes as an extension for the Pythagorean Theorem. The teacher may also use the shadow worksheet with the additional websites as a follow-up to special right triangles

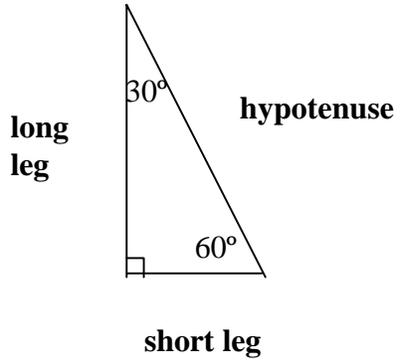
Authors:

Mary Sue France
Lake Braddock Secondary
Fairfax County, Virginia

Jennie R. Burgee
Elizabeth Seton High School
Archdiocese of Washington, Maryland

Carol L. Obermann
Wheeling Park High School
Ohio County, West Virginia

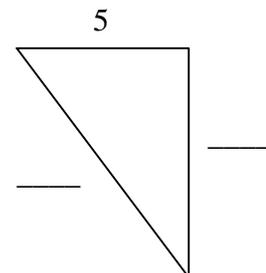
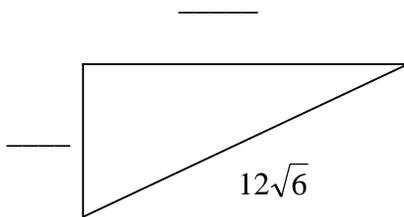
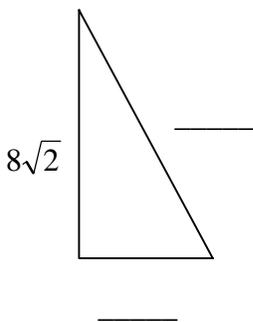
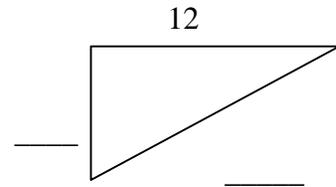
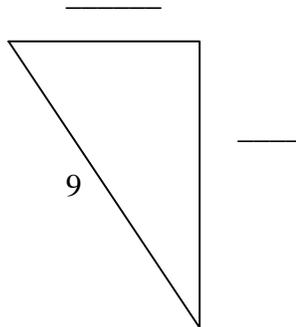
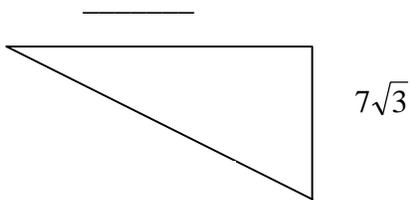
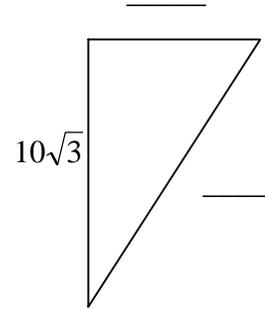
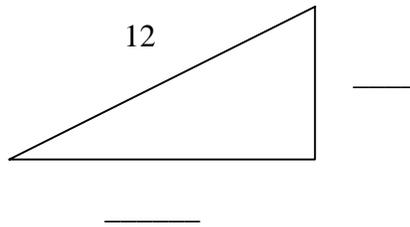
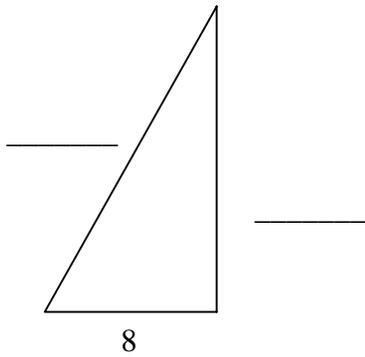
WORKSHEET #1: 30-60-90 TRIANGLES



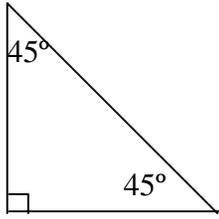
Hypotenuse = twice the short leg

Long leg = short leg * $\sqrt{3}$

Find the missing sides of the triangles. Each triangle measures 30-60-90°. Leave answers in simplified radical form.



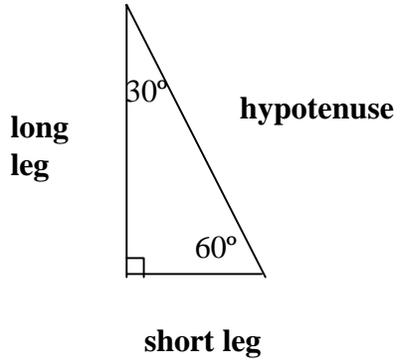
WORKSHEET #2: 45-45-90° TRIANGLES



Hypotenuse = leg * $\sqrt{2}$

Find the missing sides of the triangles. Each triangle measures 45-45-90°. Leave answers in simplified radical form.

WORKSHEET #1: 30-60-90 TRIANGLES - Key

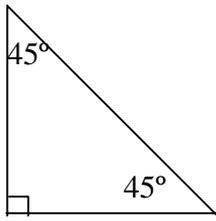


Hypotenuse = twice the short leg

Long leg = short leg * $\sqrt{3}$

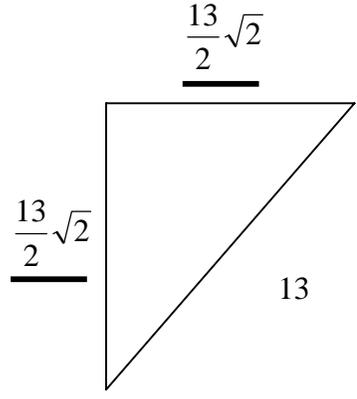
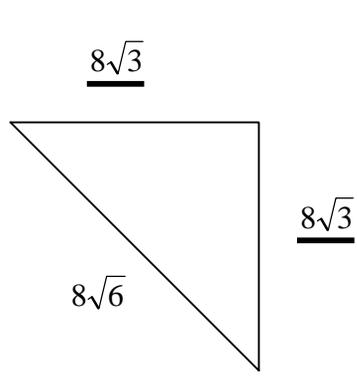
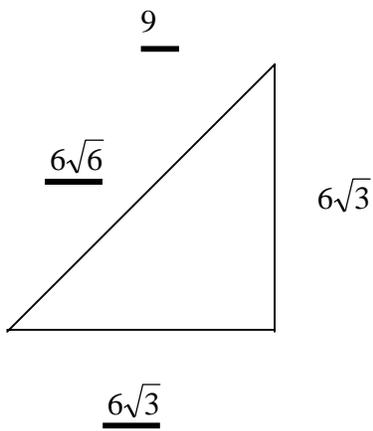
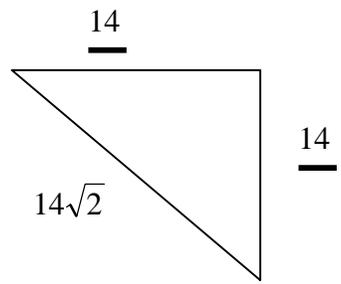
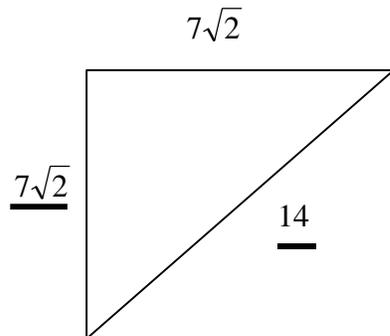
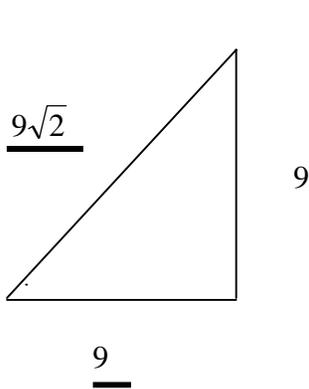
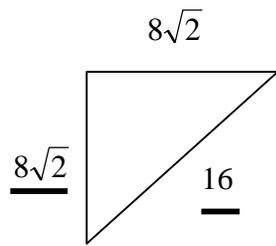
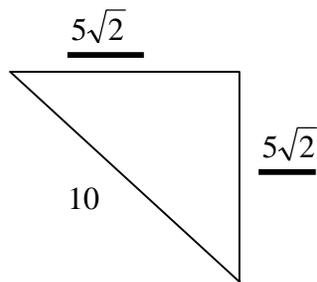
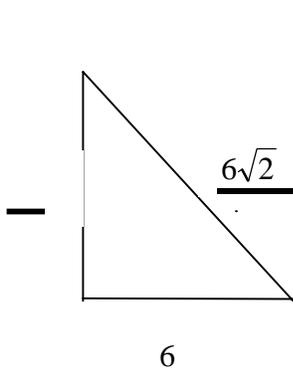
Find the missing sides of the triangles. Each triangle measures 30-60-90°. Leave answers in simplified radical form.

WORKSHEET #2: 45-45-90 TRIANGLES - Key



Hypotenuse = leg * $\sqrt{2}$

Find the missing sides of the triangles. Each triangle measures 45-45-90°. Leave answers in simplified radical form.



ACTIVITY #3—USING RIGHT TRIANGLES

As a group, you will need to do some research on a hurricane that has occurred in the past. Based on the hurricane your group selects, gather pertinent data regarding the hurricane such as when it occurred, where, and the destruction it may have caused. Your group will also need to research the plotting data to fill into the information in the table below. A good place to find this information: (<http://www.usatoday.com/weather/hurricane/whhistry.htm>)

DIRECTIONS: Based on the plotting data you find on your hurricane, fill in the columns below. The first entry should begin with the first entry in which the storm became a hurricane. You will want to record the data every 6 hours over a 60-hour period (10 entries). From the data you find on the Internet, record the date and time, longitude, latitude, and the wind speed of the hurricane at each particular time.

NAME OF HURRICANE _____
YEAR OF THE STORM _____

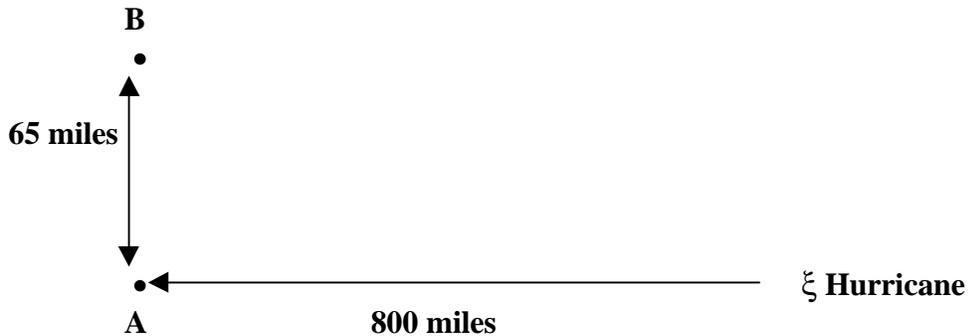
DATE/TIME	LATITUDE WITH N/S	LONGITUDE WITH E/W	MILES HURRICANE TRAVELED	WIND SPEED HURRICANE
1.			-----	
2.				
3.				
4.				
5.				
6.				
7.				
8.				
9.				
10.				

Total Miles Traveled: _____

To calculate the miles the hurricane traveled, you will need to use the longitude and latitude from each of the entries you made into your table. There is a website that will help you!! www.indo.com/distance/. Once you have retrieved the website, you will enter the **longitude with east or west and latitude with north or south** from entry #1 from your table into the “FROM” line. Then, enter the longitude and latitude from entry #2 from your table into the “TO” line. Input the miles beginning with the second row of “Miles Hurricane Traveled”. Continue down the column. This will then calculate the total miles the hurricane traveled within the 60 hours. **NOTE: MAKE SURE YOUR ENTRY IS IN MILES-NOT NAUTICAL MILES.**

ACTIVITY #4—USING RIGHT TRIANGLES

There is a ferry that travels from point A to point B continuously on a daily basis. There is a hurricane approaching point A and its path is perpendicular to the course of the ferry. The trip from point A to point B is 65 miles. The captain of the ferry has hired your group as Hurricane Consultants. As it so happens, there is a hurricane lurking in the area and is currently 800 miles east of point A. The hurricane is predicted to move west on a straight path towards point A.



USING THE DATA YOUR GROUP COLLECTED ON YOUR HURRICANE, ANSWER THE FOLLOWING QUESTIONS:

1. The hurricane is currently moving west towards point A.

PART A. Using the miles the hurricane traveled from entry #4 on your data table, calculate how far the hurricane is currently from point A. (For example, if the hurricane's distance traveled for entry #4 is 50 miles, then the hurricane is 750 miles from point A).

PART B. Calculate how far the hurricane is from point B.

2. The hurricane is currently moving west towards point A.

PART A. Using the miles the hurricane traveled from entry #8 on your data table, calculate how far the hurricane is currently from point A.

PART B. Calculate how far the hurricane is from point B.

3. Suppose the hurricane is 500 miles from point B. How far would the hurricane be from point A?

4. As the hurricane approaches closer and closer to the ferry route, the ferry company has sought out your group for advice. In a 1 page TYPED paper, they want your group to advise them when it would be wise to halt ferry operations. In this summary, you will want to find the history of the hurricane (where originated, wind speeds, how quickly it has moved, damage sustained, etc). In the paper you will want to advise when your group thinks that the ferry operation should cease. Make sure you explain your decision and support it with the facts you have gathered regarding the hurricane.

RUBRIC FOR GRADING:

3-The paper was thorough and made an argument that was substantiated with their data as to why they chose the distance that they did to halt ferry operation. The paper was grammatically correct with no spelling errors. The paper was 1 page in length and was typed.

2- The paper was thorough and made an argument that was substantiated with their data as to why they chose the distance that they did to halt ferry operation. The paper may have contained spelling and grammatical errors. The paper may or may not have been typed.

1-The paper was not thorough and did not make an argument that was substantiated with their data as to why they chose the distance that they did to halt ferry operation. The paper may or may not have been typed. It may have contained grammatical and spelling mistakes. The paper may not have been 1 page in length.

0-Students did not turn in a paper.

TEACHER NOTES/SCORING RUBRIC FOR ACTIVITY #3—USING RIGHT TRIANGLES

As a group, you will need to do some research on a hurricane that has occurred in the past. Based on the hurricane your group selects, gather pertinent data regarding the hurricane such as when it occurred, where, and the destruction it may have caused. Your group will also need to research the plotting data to fill into the information in the table below. A good place to find this information: (<http://www.usatoday.com/weather/hurricane/whhistory.htm>)

DIRECTIONS: Based on the plotting data you find on your hurricane, fill in the columns below. The first entry should begin with the first entry in which the storm became a hurricane. You will want to record the data every 6 hours over a 60-hour period (10 entries). From the data you find in the Internet, record the date and time, longitude, latitude, and the wind speed of the hurricane.

NAME OF HURRICANE _____
YEAR OF THE STORM _____

DATE/TIME	LATITUDE WITH N/S	LONGITUDE WITH E/W	MILES HURRICANE TRAVELED	WIND SPEED HURRICANE

Total Miles Traveled: _____

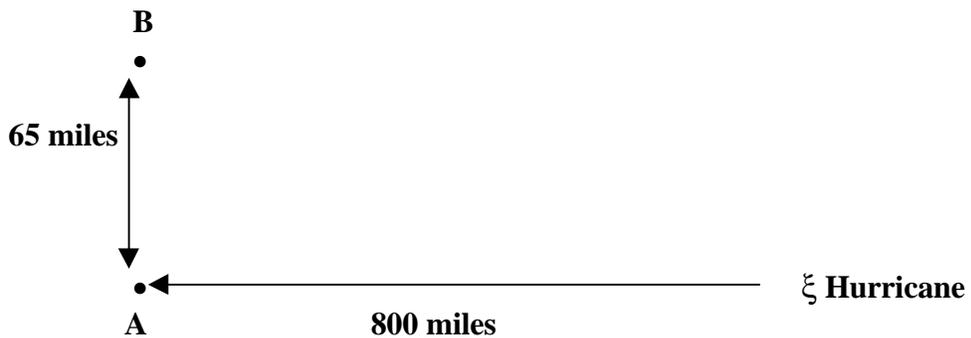
- 3- **Students have researched a hurricane on the Internet and have filled out the table with the appropriate data. The students also filled out the name and year of the hurricane and filled all rows and columns of the table except for the 4th column.**
- 2- **Students have researched a past hurricane on the Internet. The students may have left some of the data blank or incomplete.**
- 1- **Students left majority of the table blank. The name and date of the hurricane was not filled in as well.**
- 0- **Students left the entire table blank or did not turn in a paper.**

To calculate the miles the hurricane traveled, you will need to use the longitude and latitude from each of the entries you made into your table. There is a website that will help you!! www.indo.com/distance/. Once you have retrieved the website, you will enter the longitude and latitude from entry #1 from your table into the "FROM" line. Then, enter the longitude and latitude from entry #2 from your table into the "TO" line. Input the miles beginning with the second row of "Miles Hurricane Traveled". Continue down the column. This will then calculate the distance in miles the hurricane covered within the 60 hours. **NOTE: MILES SHOULD NOT BE IN NAUTICAL MILES.**

- 3- **Students were successful in using the website and converted the differences in the longitude and latitude into miles. The students were able to total the column for miles traveled for the hurricane.**
- 2- **Students were successful in converting the longitude and the latitude into miles the hurricane traveled. However, they did not have a total in the column for miles the hurricane traveled.**
- 1- **Students had difficulty converting the longitude and latitude coordinates into miles. The students may not have filled in all the rows in that column and did not calculate the total miles traveled by the hurricane.**
- 0- **Students did not complete this exercise or did not turn in a paper.**

SCORING RUBRIC FOR ACTIVITY #4—TO TRAVEL OR NOT TO TRAVEL

There is a ferry that travels from point A to point B continuously on a daily basis. There is a hurricane approaching point A and its path is perpendicular to the course of the ferry. The trip from point A to point B is 65 miles. The captain of the ferry has hired your group as Hurricane Consultants. As it so happens, there is a hurricane lurking in the area and is currently 800 miles east of point A. The hurricane is predicted to move west on a straight path towards point A.



USING THE DATA YOUR GROUP COLLECTED ON YOUR HURRICANE, ANSWER THE FOLLOWING QUESTIONS:

1. The hurricane is currently moving west towards point A.

PART A. Using the miles the hurricane traveled from entry #4 on your data table, calculate how far the hurricane is currently from point A. (For example, if the hurricane's distance traveled for entry #4 is 50 miles, then the hurricane is 750 miles from point A).

PART B. Calculate how far the hurricane is from point B.

2. The hurricane is currently moving west towards point A.

PART A. Using the miles the hurricane traveled from entry #8 on your data table, calculate how far the hurricane is currently from point A.

PART B. Calculate how far the hurricane is from point B.

TEACHER NOTE: STUDENTS NEED TO SUBTRACT TOTAL MILES HURRICANE TRAVELED FROM ENTRIES 4 AND 8 ON THEIR TABLE FROM 800 MILES. HURRICANE IS MOVING CLOSER TO POINT A.

RUBRIC FOR GRADING QUESTIONS 1 AND 2

3-Students successfully calculated the correct distance from point B to the hurricane by using the Pythagorean Theorem. Students also correctly calculated the distance to point A by subtracting miles in their entry data from 800.

2-Students used the Pythagorean Theorem to find the distance from point B to the hurricane. Students did not correctly calculate the distance to point A by subtracting miles in their entry data from 800. Also, students may not have come up with the correct answer if they made a calculation error.

1-Students did not successfully use the Pythagorean Theorem or may not have used it to calculate the distance from point B to the hurricane. They did not calculate the correct distance from the hurricane to point A.

0-Students left the problem blank.

3. Suppose the hurricane is 500 miles from point B. How far would the hurricane be from point A?

3-The students successfully used the Pythagorean Theorem to calculate the distance from point A to the hurricane.

2-The students used the Pythagorean Theorem to calculate the distance from point A to the hurricane. The students, however, may have made an error in their calculations and did not come up with the correct answer.

1-Students attempted the problem but did not use the Pythagorean Theorem to find the distance and were not able to calculate the correct answer.

0-Students left the problem blank.

4. As the hurricane approaches closer and closer to the ferry route, the Ferry Company has sought out your group for advice. In a 1 page TYPED paper, they want your group to advise them when it would be wise to halt ferry operations. In this summary, you will want to find the history of the hurricane (where originated, wind speeds, how quickly it has moved, damage sustained, etc). In the paper you will want to advise when your group thinks that the ferry operation should cease. Make sure you explain your decision and support it with the facts you have gathered regarding the hurricane.

3-The paper was thorough and made an argument that was substantiated with their data as to why they chose the distance that they did to halt ferry operation. The paper was grammatically correct with no spelling errors. The paper was 1 page in length and was typed.

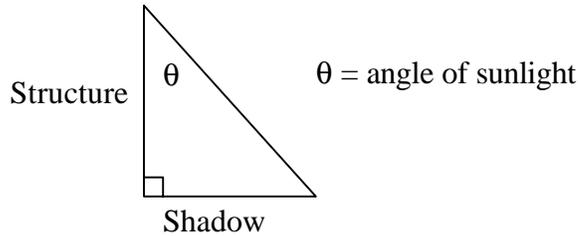
2- The paper was thorough and made an argument that was substantiated with their data as to why they chose the distance that they did to halt ferry operation. The paper may have contained spelling and grammatical errors. The paper may or may not have been typed.

1-The paper was not thorough and did not make an argument that was substantiated with their data as to why they chose the distance that they did to halt ferry operation. The paper may or may not have been typed. It may have contained grammatical and spelling mistakes. The paper may not have been 1 page in length.

0-Students did not turn in a paper.

ACTIVITY #5: SUNLIGHT AND SHADOWS

In this activity, you will find the length of the shadow cast by a structure at varying angles of sunlight. Use the Internet to find the heights of the structures. **Where applicable, give the heights of the structures without flagpoles, antennas, etc.** Then use properties of special right triangles to find the length of each shadow at the given angle. **Express all lengths to the nearest foot.**



STRUCTURE	HEIGHT IN FEET	LENGTH OF SHADOW AT EACH VALUE of θ		
		$\theta = 30^\circ$	$\theta = 45^\circ$	$\theta = 60^\circ$
1) Sears Tower, Chicago				
2) USX Tower, Pittsburgh				
3) Washington Monument				
4) Empire State Building				
5) Seattle Space Needle				

Here are some websites that you may find useful in obtaining the heights of the structures:

<http://www.nps.gov/wamo/monument/monument.htm>

<http://www.worldstallest.com/hot500.html>

<http://www.imsa.edu/~mozart/fun/needle.html>

TEACHER NOTES FOR SUNLIGHT AND SHADOWS

Suggested answers are given below. Answers may vary slightly depending upon the source used to find the heights of the structures.

STRUCTURE	HEIGHT IN FEET	LENGTH OF SHADOW AT EACH VALUE of θ		
		$\theta = 30^\circ$	$\theta = 45^\circ$	$\theta = 60^\circ$
1) Sears Tower, Chicago	1450	837	1450	2511
2) USX Tower, Pittsburgh	841	486	841	1457
3) Washington Monument	555	320	555	961
4) Empire State Building	1250	722	1250	2165
5) Seattle Space Needle	605	349	605	1048

Here are some websites that students may find useful in obtaining the heights of the structures:

<http://www.nps.gov/wamo/monument/monument.htm>

<http://www.worldstallest.com/hot500.html>

<http://www.imsa.edu/~mozart/fun/needle.html>

As a hands-on extension to this activity, you may want to check out the website

<http://www.kyes-world.com/pythagor.htm>

This site contains “Pythagoras’ Playground,” which offers activities using student-made quadrants for finding heights of buildings, trees, etc.